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## Optimal Fiscal Policy in the Design of Social Security Reforms

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# Optimal Fiscal Policy in the Design of Social Security Reforms

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## ABSTRACT

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The quantitative macroeconomics literature has documented that in the basic Overlapping Generations model a privatization of the social security system, going from a Pay-As-You-Go to a Fully Funded system, generates large long run welfare gains at the cost of substantial welfare losses for initial generations. We propose an alternative to previous literature. In this paper we maximize over the entire policy space, following the optimal fiscal policy approach, rather than comparing alternative policy paths one to one. That is, policies are chosen as part of the optimal design of a social security privatization in a Pareto improving way. The government decides endogenously how to finance the implicit social security liabilities and compensate the initial generations alive during the transition. In contrast with previous analysis the resulting allocation, by construction, lies on the constrained Pareto frontier. We find that the optimal design of reforms exhibits sizeable welfare gains, arising because of the reduction in labor supply distortions. In contrast, the welfare gains coming from the reduction of savings distortions are relatively small.

Keywords: Optimal Taxation, Ramsey Policies, and Constrained Optima

JEL Classification: E62, H21

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# 1. Introduction

Efficiency considerations have often been used as arguments for reforming public Social Security systems, usually of a Pay-As-You-Go (PAYG) nature, in favor of Fully Funded systems (FF). As a consequence, research on the quantitative evaluation of social security reforms to assess their efficiency gains has been one of the main topics in this area.<sup>1</sup> The standard large scale Overlapping Generations (OG) model predicts very large efficiency and welfare gains in the long run from funding social security. Nevertheless, in most studies the consideration of the transitional dynamics has lead researchers to conclude that, despite the potential large welfare gains, the privatization of the social security system involves substantial welfare losses along the transition. In this paper we propose an alternative approach: the use of optimal fiscal policy tools in order to maximize over the policy space instead of comparing parametric reforms one to one. By using such a strategy we identify constrained Pareto efficient policies that allow financing the transition from a PAYG to a FF system in a Pareto improving way. In addition, the approach allows going beyond social security reform (or privatization) and explore the welfare gains that come from the relaxation of the constraints on tax instruments in a broader tax reform.

Building on the seminal work of Auerbach and Kotlikoff (1987), there are several papers that study the transition associated to a social security privatization and find substantial efficiency and welfare gains in the long run.<sup>2</sup> In particular Huang, Imrohoroglu and Sargent (1997) show that a complete or partial privatization implies large short-run welfare losses, which cannot be compensated with the long-run gains. Conesa and Krueger (1999) show that in the presence of uninsurable labor income uncertainty the welfare losses of the initial cohorts are even larger, because the unfunded social security system provides partial insurance. Kotlikoff, Smetters and Walliser (1999) analyze different types of transitions and find that transition generations experience a 1 to 3 percent welfare decline, while future generations experience gains that are close to 20 percent. Using a different approach, Feldstein and Samwick (1998) find smaller but still positive transition costs. Conesa and Garriga (2003) show that eliminating compulsory retirement rules with the privatization can substantially reduce

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<sup>1</sup> Feldstein and Liebman (2002) summarize the discussion on transition to investment-based systems, analyzing the welfare effects and the risks associated to such systems.

<sup>2</sup> Theoretical frameworks that introduce dynastic considerations within the life-cycle framework, such as some sort of intergenerational links as in Fuster (1999) or Fuster, Imrohoroglu and Imrohoroglu (2004), might imply that the efficiency gains are much more moderate or even inexistent.

the welfare losses of the initial generations alive, but yet these are still substantial. Finally, Prescott (2005) and Joines (2005) identify a policy path such that no cohorts are worse off. In general all these alternative plans for social security reforms have been proposed and compared one to one. However, it is possible to do better, in particular, tax plans that come from a well-defined welfare maximization problem. This represents our main contribution to the existing literature on the macroeconomic and welfare implications of social security reforms. We provide an environment in which the reforms are endogenously chosen by the fiscal authority given status quo constraints. In doing so we go beyond identifying a particular policy that would do the job, and we identify policies lying on the constrained (by the use of distortionary taxation) Pareto frontier.<sup>3</sup>

Notice that if there were no distortions and the economy was dynamically efficient it is not feasible to redistribute resources across generations in a Pareto improving way. This classic result goes back to Diamond (1965) and Gale (1973), who studied the “Classical case” as compared to the “Samuelson case” of dynamic inefficiency. A social security system is an intergenerational redistribution device, and hence changing the system itself cannot generate welfare gains. Alternatively, if the financing of social security is distortionary, Homburg (1990) and Rangel (1997) use a two period overlapping generations model to show that Pareto improvements are possible by reducing labor supply distortions. In a similar spirit Feldstein (1995, 1998) shows that two conditions are required in order to increase the present value of consumption of all generations. First, the return on capital must exceed the implicit return in the unfunded system. Second, the marginal product of capital exceeds the social discount rate. Our benchmark economy will satisfy both conditions by construction. Necessarily, the presence of distortions in our environment is what allows us to design Pareto improving reforms, and thus the Ramsey approach as pioneered by Escolano (1992), Erosa and Gervais (2002) and Garriga (1999) seems the natural approach.

The relevant aspect in our exercise is how to generate and redistribute the surplus generated by the minimization of distortions in order to engineer a Pareto improving social security reform. Alternatively, one could take the social security as an institutional constraint on the policy maker and then the same approach could be used to

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<sup>3</sup> By assumption lump-sum taxes are not allowed, as usual in the Ramsey approach to optimal fiscal policy. In contrast to the Ramsey approach, Golosov, Kocherlakota and Tsyvinski (2003) pioneered an alternative approach to dynamic optimal fiscal policy based on the existence of informational asymmetries. In their analysis the set of tax instruments is endogenously determined by the information set available to the fiscal authority.

determine the optimal financing of such a system.<sup>4</sup>

We abstract from a large and growing literature on positive theories of social security, starting with Samuelson (1975) focusing on dynamic inefficiency, Feldstein (1985) and Diamond (2004) pointing at myopic behavior, Cooley and Soares (1999), Boldrin and Rustichini (2000), Forni (2005) stressing the political economy aspects, Krueger and Kubler (2006) focusing on intergenerational risk-sharing or Boldrin and Montes (2004) focusing on intergenerational contracts.

In our benchmark economy there is survival uncertainty and we assume away annuity markets, so that the only efficiency role of a social security system is to partially substitute for this missing market.

Demographic considerations also play an important role in the social security debate. However, in order to focus on efficiency considerations we abstract from demographic changes. For example, see De Nardi, Imrohoroglu and Sargent (1999) and Jeske (2003) for a quantitative evaluation of the impact of demographic projections on the U.S. social security imbalances.

The quantitative analysis of optimal fiscal policy in OG economies was pioneered by Escolano (1992), whereas the theoretical properties have been recently considered by Erosa and Gervais (2002) and Garriga (1999). Following these papers, we will show the importance of different sets of tax instruments for generating the results. Notice that whenever we allow for age-specific capital income taxes this will allow the fiscal authority to implement annuities. Our contribution relative to these papers is not how to define optimal fiscal policy in an OG framework, but rather how to use this framework to identify the relevant margins that allow for Pareto improving social security reforms (something that has proved very hard so far in the social security reform quantitative literature).

We will first analyze a benchmark case where only compensatory transfers to the initial old, government debt and time varying labor income taxes are allowed. Later we will investigate how policy recommendations and welfare would change if we consider tax reforms that allow for a larger set of instruments (capital income taxes and age-dependence of taxes).

Our main conclusions are the following:

1. There exists a Pareto neutral reform, making explicit the implicit debt of social security and leaving all distortions unchanged, but it is possible to do better than that by

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<sup>4</sup> Conesa and Garriga (2007) follows this alternative scenario and solves for the optimal financing of the existing level of pensions when the economy faces an adverse transitory demographic shock.

reducing distortions.

2. Optimal social security reforms consist of providing compensatory transfers to the initial old (almost as large as their social security entitlements) financed with debt and lowering labor income taxes on impact in order to increase them later on.
3. The relaxation of the constraints on tax instruments shifts the distribution of welfare gains between present and future cohorts. The ability to discriminate labor income taxes across cohorts substantially reduces the need to resort to compensatory transfers to the initial old generations.
4. Introducing capital income taxes in the analysis generates some additional welfare gains, but it usually implies very large non-smooth changes in capital income taxes that call into question its relevance as actual policy options.

The rest of the paper is organized as follows. Section 2 describes the economic environment under the Status Quo policies. Section 3 describes how to view PAYG social security as an implicit debt and the neutrality of making explicit this debt. Section 4 discusses how the benchmark economy is parameterized. Section 5 shows how to specify the government problem. Section 6 discusses the results in our benchmark scenario: the government can only use compensatory transfers to the initial old, age-independent labor income taxes, and debt. Section 7 discusses how the conclusions change if we relax the constraints imposed on the set of tax instruments. Section 8 concludes. All the references are in Section 9.

## 2. The Status Quo Economic Environment

### Households

The economy is populated by a measure of households who live for a maximum of  $I$  periods and grows at rate  $n$ . We denote by  $\varphi_i$  the conditional survival probability, i.e.  $\varphi_i = \text{Prob}(\text{alive at age } i+1 \mid \text{alive at age } i)$ . Therefore, let  $s_1 = 1$ ,  $s_i = \varphi_1 \cdot \varphi_2 \cdot \dots \cdot \varphi_{i-1}$  be the unconditional probability of being alive at age  $i$ . We will denote  $\mu_i$  the measure of households of age  $i$ , computed as:

$$\mu_i = \frac{1}{1+n} \varphi_i \mu_{i-1}$$

Preferences of a household born in period  $t$  depend on the stream of consumption and leisure this household will enjoy. Thus, expected utility is given by:

$$U(c^t, l^t) = \sum_{i=1}^I s_i \beta^{i-1} u(c_{i,t+i-1}, 1 - l_{i,t+i-1}) \quad (1)$$

Each household owns one unit of time in each period that they can allocate for work or leisure. One unit of time devoted to work by a household of age  $i$  translates into  $\varepsilon_i$  efficiency units of labor in the market. These households compulsory retire in period  $i_r$ . Finally, since there is survival uncertainty and we assume away annuity markets, we will assume that the government collects the assets of the deceased. This assumption is done for the following reasons. First, notice that the taxation of transfers such as unintended bequests is non-distortionary, so a benevolent government would choose to tax them away. Also, our assumption guarantees that the Ramsey planner will not have access to additional non-distortionary sources of taxation (taxing away accidental bequests) not present in the benchmark economy.

### Technology

The production possibility frontier is given by an aggregate production function  $Y_t = F(K_t, A_t L_t)$ , where  $K_t$  denotes the capital stock in period  $t$  and  $L_t = \sum_{i=1}^I \mu_i \varepsilon_i l_{i,t}$  is aggregate labor measured in efficiency units. Aggregate labor efficiency,  $A_t$ , grows at an exogenous rate of technological progress  $x$ . We assume the function  $F$  displays constant returns to scale, is monotonically increasing, strictly concave and satisfies the Inada conditions. The capital stock depreciates at a constant rate  $\delta$ .

The resource constraint is given by:

$$\sum_{i=1}^I \mu_i c_{i,t} + (1+x)(1+n)k_{t+1} - (1-\delta)k_t + g_t \leq f\left(k_t, \sum_{i=1}^I \mu_i \varepsilon_i l_{i,t}\right)$$

where consumption, capital and government expenditure are deflated by the growth rate of output in a balanced growth path  $(1+x)(1+n)$ .

### Government

The government influences this economy through the social security and the general budget. For simplicity we assume that in the benchmark economy these two programs operate with different budgets. Then, pensions ( $p_t$ ) are financed through a payroll tax

$(\tau_t^p)$  and the social security budget is balanced. On the other hand, the government collects consumption taxes  $(\tau_t^c)$ , labor income taxes  $(\tau_t^l)$ , capital income taxes  $(\tau_t^k)$  and issues public debt  $(b_t)$  in order to finance an exogenously given stream of government consumption  $(g_t)$ .

Thus the government budget constraints are given by:

$$\tau_t^p w_t \sum_{i=1}^{i_r-1} \mu_i \varepsilon_i l_{i,t} = p_t \sum_{i=i_r}^I \mu_i \quad (2)$$

$$\tau_t^c \sum_{i=1}^I \mu_i c_{i,t} + \tau_t^l (1 - \tau_t^p) w_t \sum_{i=1}^{i_r-1} \mu_i \varepsilon_i l_{i,t} + \tau_t^k r_t \sum_{i=1}^I \mu_i a_{i,t} + \sum_{i=1}^I \mu_{i-1} (1 - \phi_{i-1}) a_{i,t} + (1+x)(1+n)b_{t+1} = g_t + (1+r_t)b_t \quad (3)$$

Notice that all variables are deflated by the growth rate  $(1+x)(1+n)$ .

### Market arrangements

We assume there is a single representative firm that operates the aggregate technology taking factor prices as given. Households sell an endogenously chosen fraction of their time as labor  $(l_{i,t})$  in exchange for a competitive wage of  $w_t$  per efficiency unit of labor. They rent their assets  $(a_{i,t})$  to firms or the government in exchange for a competitive price  $(r_t)$ , and decide how much to consume and save out of their disposable income. The sequential budget constraint for a working age household is given by:

$$(1 + \tau_t^c) c_{i,t} + (1+x) a_{i+1,t+1} = (1 - \tau_t^l) (1 - \tau_t^p) w_t \varepsilon_i l_{i,t} + (1 + (1 - \tau_t^k) r_t) a_{i,t} \quad (4)$$

Upon retirement households do not work and receive a public pension in a lump-sum fashion. Their budget constraint is:

$$(1 + \tau_t^c) c_{i,t} + (1+x) a_{i+1,t+1} = (1 - \tau_t^l) p_t + (1 + (1 - \tau_t^k) r_t) a_{i,t} \quad (5)$$

The alternative interpretation of a mandatory retirement rule is to consider different labor income tax rates for individuals of ages above and below  $i_r$ . In particular, a confiscatory tax on labor income beyond age  $i_r$  is equivalent to compulsory retirement. Both formulations yield the same results. However, when we study the optimal policy we prefer this alternative interpretation since it considers compulsory retirement as just one more distortionary tax that the fiscal authority can optimize over.

It is important to note that in the way the environment is written consumers do not see any connection between their social security contributions while working and their



future social security pensions. If consumers would establish such a link and hours worked would be chosen taking into account its effect on future pensions, then the distortionary impact of social security would be smaller. See Feldstein and Liebman (2002) for a discussion of this issue.

Finally notice we assumed away annuity markets, so that social security plays a role as partial insurance.

**Definition 1:** A market equilibrium in the status quo economy is a sequence of prices and allocations such that: i) consumers maximize utility subject to their corresponding budget constraints given the equilibrium prices, ii) firms maximize profits given prices, iii) the government and the social security budgets are balanced, and iv) markets clear and feasibility.

### **3. PAYG Social Security as Implicit Debt and Pareto Neutral Reforms**

An unfunded social security system is an intergenerational redistribution scheme, or equivalently an implicit debt scheme. The young provide resources through contributions that are used to finance the benefits of the retired. Contributions made by the young generate an entitlement to a future benefit upon retirement, which constitutes an implicit debt of the social security administration towards them. Upon retirement, these new retirees sell their claims to social security to the new cohorts of workers.

Consequently, a social security privatization only amounts to making explicit the implicit debt. Diamond (1965) and Gale (1973) showed that if the economy is dynamically efficient and the labor supply is perfectly inelastic, then it is not possible to redistribute resources across generations in a Pareto improving way. As a result, in their framework, a PAYG social security system cannot be transformed into a fully funded system in a way that makes every cohort better off. Next, we show that this result can be extended to a more general setup. The idea is very simple, it amounts to show that an equilibrium for an economy with an unfunded social security system is equivalent to an equilibrium of an economy with funded social security where the implicit debt is made explicit.

**Proposition 1:** Let  $\{\hat{\tau}_t^c, \hat{\tau}_t^l, \hat{\tau}_t^k, \hat{\tau}_t^p, \hat{p}_t, \hat{B}_t\}_{t=1}^\infty$  be a specific fiscal policy of an economy with unfunded social security, and let  $\{(\hat{c}_{i,t}, \hat{l}_{i,t})_{i=1}^I, \hat{K}_t\}_{t=1}^\infty$  be the associated allocation with asset distribution  $\{(\hat{a}_{i,t})_{i=1}^I\}_{t=1}^\infty$ . Then, there exists a fiscal policy with funded social security,  $\{\hat{\tau}_t^c, \hat{\tau}_t^l, \hat{\tau}_t^k, \hat{\tau}_t^p, 0, \tilde{B}_t\}_{t=1}^\infty$ , and a distribution of assets  $\{(\tilde{a}_{i,t})_{i=1}^I\}_{t=1}^\infty$  such that  $\{(\hat{c}_{i,t}, \hat{l}_{i,t})_{i=1}^I, \hat{K}_t\}_{t=1}^\infty$  is the equilibrium allocation corresponding to policy  $\{\hat{\tau}_t^c, \hat{\tau}_t^l, \hat{\tau}_t^k, \hat{\tau}_t^p, 0, \tilde{B}_t\}_{t=1}^\infty$ .

**Proof:** First, since we leave all tax rates (including social security contributions<sup>5</sup>) and prices as in the benchmark economy, clearly the Euler and Labor Supply conditions of the consumer's problem are satisfied for the same allocation:

$$(1+x)u_{\hat{c}_{i,t}} = \varphi_i \beta u_{\hat{c}_{i+1,t+1}} \left[ 1 + (1 - \tau_{t+1}^k) \hat{r}_{t+1} \right] \quad (6)$$

$$-\frac{u_{\hat{l}_{i,t}}}{u_{\hat{c}_{i,t}}} = \frac{(1 - \tau_t^l)(1 - \tau_t^p)}{1 + \tau_t^c} \hat{w}_t \varepsilon_i \quad (7)$$

Next, we construct a new sequence of assets in the following way:

$$\begin{aligned} \tilde{a}_{I,t} &= (1 + \tau_t^c) \hat{c}_{I,t} / \left[ 1 + (1 - \tau_t^k) \hat{r}_t \right] \\ \tilde{a}_{i,t} &= \frac{(1 + \tau_t^c) \hat{c}_{i,t} + (1+x) \tilde{a}_{i+1,t+1} - (1 - \tau_t^l)(1 - \tau_t^p) \hat{w}_t \varepsilon_i \hat{l}_{i,t}}{1 + (1 - \tau_t^k) \hat{r}_t}, i = I-1, I-2, \dots, 1 \end{aligned} \quad (8)$$

Here notice that the process has to be done backwards with respect to time, but this is not a problem since we know all the consumption/labor allocations from now to infinity.

This new sequence of assets has been constructed such that the retirement pensions are set to zero.<sup>6</sup> Notice then that  $\tilde{a}_{1,t}$  is not zero, in fact it will be equal to the net present value of future pensions received under the PAYG regime since the intertemporal budget constraint has to be the same in order to support the same consumer allocation. This could be decentralized in the form of a lump-sum tax cut for the youngest cohort. If we sum this new asset distribution across cohorts and time, and subtract the original capital stock we obtain a different level of government debt,  $\tilde{B}_t > \hat{B}_t$ . The difference

<sup>5</sup> Notice that now the distinction between labor income taxes (going to the general budget) and payroll taxes (to finance pensions) is meaningless, since there are no pensions to finance anymore.

<sup>6</sup> Figure 2 will display the original asset distribution (labeled in the figure as “With PAYG”) and the new one constructed in the way we just explained (labeled in the figure as “Implicit Assets”), for our parameterized economy.

between this new level of debt and the original one is the implicit debt of the social security system. The implicit debt of the social security system is a constant fraction of output along a balanced growth path, but might change along transitional dynamics since prices and pensions change over time.

Since the allocation is the same as in the original equilibrium feasibility is satisfied. Walras' Law guarantees that the Government Budget Constraint holds. Finally, notice that since the aggregates have not changed, the equilibrium prices for the economy with the funded system and for the original economy with an unfunded system are the same. Hence, we do have a competitive equilibrium with funded social security. ■

An immediate application of the equivalence result is that it would be straightforward to engineer a Pareto neutral social security privatization. First, notice that the way the implicit assets are constructed the only thing we would be doing is giving to all currently alive consumers a lump sum transfer equal to the net present value of their future social security payments, i.e. their social security entitlements. Then, by construction their intertemporal budget constraint is not affected and their allocations would not change.

Furthermore, if every newborn generation would be given a transfer equal to the net present value of social security payments at birth (the level of assets corresponding to age 1), the intertemporal budget constraint would not change and therefore it would be optimal to choose the same allocation of consumption and leisure, together with a sequence of assets equal to the one just constructed.

Neither the initial old nor any subsequent newborn generation would change behavior relative to the original allocation under the PAYG regime. Moreover, since the allocation is feasible and the consumers' budget constraints are satisfied, the government budget constraint is also satisfied. We have just shown how to change the direction of intergenerational transfers without affecting allocations or welfare, thus making explicit the implicit debt of social security.

Since intergenerational redistribution cannot itself generate Pareto improvements in a dynamically efficient economy, Pareto improvements are only possible if and only if there exist distortions in social security financing or in the general fiscal system. This issue has been addressed in Homburg (1990) and Rangel (1997) using a two period overlapping generations model.

In what follows we show how to generate substantial efficiency gains both in the short-run and in the long-run. We will perform such an exercise in a parameterized version of

a standard large-scale OG framework. Then, we will maximize over policies in an environment where both transition cohorts and future newborns are better off than in the benchmark economy and discuss specific policy recommendations. In addition, we explore the welfare impact of allowing for a larger set of fiscal instruments so we can measure the quantitative importance of minimizing distortions on various margins.

It is important to note here that in the standard social security reform literature pensions and social security contributions are eliminated over time following some arbitrary way, and then the macroeconomic and welfare implications are analyzed. Effectively, most of this type of exercises implied some partial or complete default on promises (equivalent to a default on the implicit debt of social security), which was at the root of the large welfare losses of transition cohorts. In our exercise this is ruled out, since we maximize over policies under the constraint that everybody is made better off.

## 4. Parameterization of the Status Quo Economy

### Demographics

One period in the model is the equivalent of 5 years. Given our choice of period we assume households live up to a maximum of 14 periods, so that the economically active life of a household starts at age 20 and we assume that households live at most to age 89. In the benchmark economy households retire in period 10 (equivalent to age 65 in years).

Survival probabilities,  $(\phi_i)$ , are taken from Bell and Miller (2002), with the assumption that households die with probability 1 when reaching age 90.

Finally, we assume that the mass of newborn households (and hence the total mass of households) grows at an annual rate  $n = 1.1\%$ .

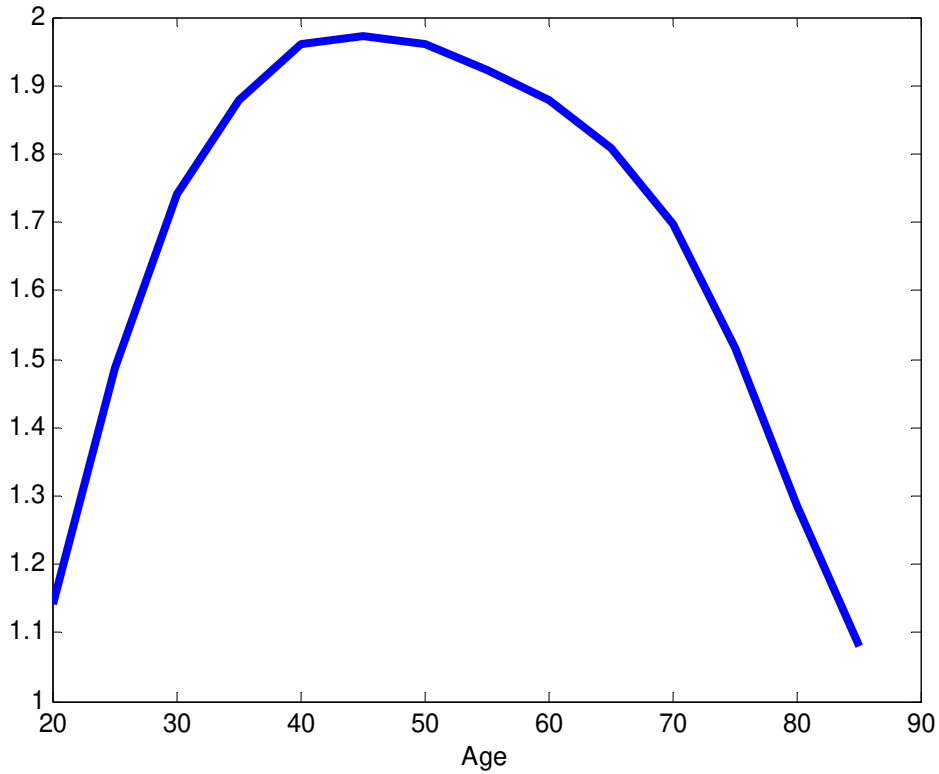
### Endowments

The only endowment households have is their efficiency units of labor at each period. These are taken from the Hansen (1993) estimates, conveniently extrapolated to the entire lifetime of households.<sup>7</sup>

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<sup>7</sup> In order to avoid sample selection biases we assume that the rate of decrease of efficiency units of labor after age 65 is the same as in the previous period.

**Figure 1: Age-Profile of Efficiency Units of Labor from Hansen (1993)**



### **Government**

We assume that in the benchmark economy the government runs two completely independent budgets. One is the social security budget that operates on a balanced budget. The payroll tax is taken from the data and is equal to 10.5%, which is the Old-Age and Survival Insurance, OASI (excluding Medicare and Disability Insurance). Our assumptions about demographics (implying a ratio of population over 65 to working age of 24%) together with the balanced budget condition directly determine the amount of the public retirement pension. It will be 43% of the average gross labor income (51% relative to labor income net of taxes).

The level of government consumption is exogenously given. It is financed through a consumption tax, set equal to 5%, a marginal tax on capital income equal to 33% and a marginal tax on labor income net of social security contributions equal to 16%. We use the methodology proposed by Mendoza, Razin and Tesar (1994) to estimate these effective tax rates. The effective distortion of the consumption-leisure margin is given by  $(1-\tau^l)(1-\tau^p)/(1+\tau^c)=1-0.3$ , yielding an effective tax of 30%.

The government issues public debt in order to satisfy its sequential budget constraint.

### Calibration: Functional Forms

Households' preferences are assumed to take the form:

$$\sum_{i=1}^I s_i \beta^{i-1} \frac{(c_i^\gamma (1-l_i)^{1-\gamma})^{1-\sigma}}{1-\sigma} \quad (9)$$

where  $\beta > 0$  represents the discount rate,  $\gamma \in (0,1)$  denotes the share of consumption on the utility function, and  $\sigma > 0$  governs the concavity of the utility function. The implied intertemporal elasticity of substitution in consumption is equal to  $1/(1-(1-\sigma)\gamma)$ .

Technology has constant returns to scale and takes the standard Cobb-Douglas form:

$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$  where  $\alpha$  represents the capital income share. We assume that labor efficiency,  $A_t$ , grows at  $x = 1.75\%$  a year.

### Calibration: Empirical Targets

We define aggregate capital to be the level of Fixed Assets in the BEA statistics. Therefore, our calibration target will be a ratio  $K/Y=3$  in yearly terms. Also, computing the ratio of outstanding (federal, state and local) government debt to GDP we get the following ratio  $B/Y=0.5$  in yearly terms. Depreciation is also taken from the BEA statistics, it is a fraction of 12% of GDP. Another calibration target is an average of 1/3 of the time of households allocated to market activities. We will choose a curvature parameter in the utility function consistent with a consumption intertemporal elasticity of substitution of 0.5 (around empirical values obtained in the literature). Government consumption is fixed to 18.6% of output, as observed in the data. Finally, the capital income share is measured from the national accounts.

### Calibration Results

In order to calibrate our economy we proceed as follows. First, we fix the curvature parameter in the utility function to be  $\sigma = 4$  and the capital share in the production function  $\alpha = 0.34$ . Then the discount factor  $\beta = 1.0375$  is chosen to match a wealth to output ratio of 3.5, and the consumption share  $\gamma = 0.31$  is chosen in order to match an average of 1/3 of time devoted to working in the market economy. The depreciation rate is chosen so that in equilibrium depreciation is 12% of output, as in the data.

Notice that  $\sigma = 4$  and  $\gamma = 0.31$  together imply a consumption intertemporal elasticity of substitution of 0.5 (CRRA of 2).

Table 1 summarizes the parameters chosen and the empirical targets that are more related to them.

<b>Table 1: Calibration Targets and Parameter Values</b>					
Empirical Targets	A/Y	IES	Av.Hours	wN/Y	Dep./Y
Empirical Values	3.5	0.5	1/3	0.66	0.12
Parameters	$\beta$	$\sigma$	$\gamma$	$\alpha$	$\delta$
Calibrated Values	1.0375	4	0.31	0.34	0.0436

Using the empirical tax rates and ratio of government consumption to GDP, we derive from the government budget constraint an implied equilibrium government debt of 50% of output. This figure is consistent with the average figure in the data. Therefore, the capital/output ratio is 3 as desired.

Given this parameterization, social security payments in the benchmark economy amount to 6.9% of GDP.

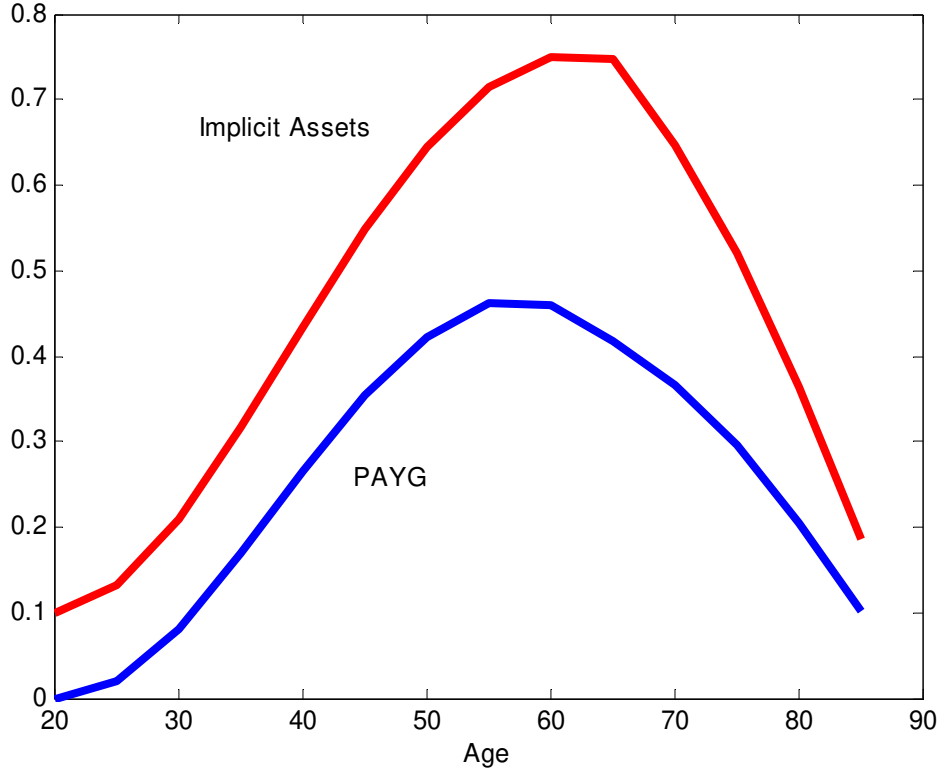
### **Social Security as Implicit Debt: a Quantitative Measurement**

Given the model parameterization, we first calculate the implicit debt of the social security system.

Consider an alternative decentralization for the same Steady State allocation associated to the parameterized economy just described. We construct it following exactly the same steps as in Proposition 1. The newly constructed asset allocation is the sequence  $\{\tilde{a}_i\}_{i=1}^I$ , labeled as “Implicit Assets” in Figure 2. Notice that if we integrate this new asset allocation and subtract the Steady State capital stock we find a new level of debt,  $\tilde{B}$ . The difference between this new level of debt and the original one is the implicit debt of social security (the sum of the net present value of future pensions of all cohorts), which in our parameterization is 2.2 times GDP.

Notice that here the assumption of complete markets might be important. If a particular age cohort is borrowing constrained, then a change of the direction of intergenerational redistribution could alleviate its constraint, changing its behavior and therefore prices would change. Hence, in order to satisfy the equivalence of allocations the lump sum payment should be made in a period where the individual is not borrowing constrained.

**Figure 2: Implicit Assets of the PAYG Social Security System**



## 5. Specifying the Ramsey Problem

We assume that in period  $t=1$  the economy is in a steady state with a PAYG social security system, and no reform has been anticipated by any of the agents in the economy. The expected utility for each generation associated to remaining in an economy with the existing PAYG social security system is given by:

$$\bar{U}_j = \sum_{i=j}^I \frac{s_i}{s_j} \beta^{i-j} u(\hat{c}_i, 1 - \hat{l}_i) \quad (10)$$

where  $\hat{c}_i, \hat{l}_i$  are steady state allocations of generation  $j$ .

At the beginning of period 2, the government implements a FF social security system and gives a one-period lump-sum transfer to all the initial generations alive who have contributed to the old PAYG system. The total amount of optimally chosen transfers is financed issuing new debt. To maximize the size of the welfare gains we let the government choose the level of debt issued and the optimal tax mix to finance the newly issued debt and the pre-existing level of government expenditure.



The government objective function is a utilitarian welfare function of all future newborn individuals, where the relative weight that the government places between present and future generations is captured by the geometric discount factor  $\lambda \in (0,1)$ . Formally,

$$\sum_{t=2}^{\infty} \lambda^{t-2} U(c^t, l^t) \quad (11)$$

where  $U(c^t, l^t)$  represents lifetime utility of generation born in period  $t$ .

We use the primal approach to optimal taxation as discussed in Atkinson and Stiglitz (1980). This approach is based on characterizing the set of allocations that the government can implement for a given policy. The government chooses the optimal tax burden taking into account the decision rules of all individuals in the economy, and the effect of their decisions on market prices.

Therefore, the government problem amounts to maximizing its objective function over the set of implementable allocations together with the status quo constraints.<sup>8</sup> From the optimal allocations we can decentralize the economy finding the prices and the tax policy associated to the social security reform. The derivation of the set of implementable allocations is very similar to the formulations derived by Erosa and Gervais (2002) and Garriga (1999). Since there are some differences because of the presence of survival uncertainty, we show how to derive them in the Appendix. It is worth pointing out that the implementability constraints do not depend on the availability or not of annuity markets, since annuities are equivalent to age-dependent capital income taxes (making the return on savings contingent on age). Nevertheless, the absence or not of annuities will matter when we impose the constraint of age-independence of capital income taxes on the Ramsey problem.

Conditional on our choice of weights placed on different generations<sup>9</sup>, the set of constrained efficient allocations can be obtained through the following maximization problem:

---

<sup>8</sup> Throughout the paper we assume that the government can commit to its policies ignoring time consistency issues. Clearly, this is an important restriction that affects the results. The analysis of a time consistent reform goes beyond the scope of this paper.

<sup>9</sup> We are just identifying one Pareto efficient reform, but it is clearly not unique. Placing different weights on generations or the initial old would generate a different distribution of welfare gains across agents.

$$\max \sum_{t=2}^{\infty} \lambda^{t-2} U(c^t, l^t)$$

$$s.t. \sum_{i=1}^I \mu_{i,t} c_{i,t} + (1+x)(1+n)k_{t+1} - (1-\delta)k_t + g_t \leq f\left(k_t, \sum_{i=1}^I \mu_{i,t} \varepsilon_i l_{i,t}\right), \quad t \geq 2 \quad (12)$$

$$\sum_{i=1}^I s_i \beta^{i-1} (c_{i,t+i-1} u_{c_{i,t+i-1}} + l_{i,t+i-1} u_{l_{i,t+i-1}}) = 0, \quad t \geq 2 \quad (13)$$

$$\sum_{i=j}^I \frac{s_i}{s_j} \beta^{i-j} \left[ c_{i,i-j+2} u_{c_{i,i-j+2}} + l_{i,i-j+2} u_{l_{i,i-j+2}} \right] = \frac{u_{c_{j,2}}}{1+\tau_2^c} \left[ (1+(1-\tau_2^k)r_2) \bar{a}_{j,2} + t_{j,2} \right], \quad j = 2, \dots, I \quad (14)$$

$$\sum_{i=j}^I \frac{s_i}{s_j} \beta^{i-j} u(c_{i,i-j+2}, 1-l_{i,i-j+2}) \geq \bar{U}_j, \quad j = 2, \dots, I \quad (15)$$

$$U(c^t, l^t) \geq \bar{U}_1, \quad t \geq 2 \quad (16)$$

$$\text{Given } K_2, \tau_2^k, \tau_2^c$$

where the choice variables are all the allocations and potentially a compensatory transfer  $t_{j,2} \geq 0$  to each of the initial generations alive  $j$ , at the initial period of the reform.

Constraint (12) is the standard resource constraint. Constraint (13) is the implementability constraint for each generation born after the reform is announced and implemented. Constraint (14) represents the implementability constraints for those generations alive at the beginning of the reform, where  $\tau_2^k$  and  $\tau_2^c$  are the benchmark taxes on capital income and consumption which are taken as given, and  $\bar{a}_{j,2}$  are the initial asset holdings of generation  $j$ . Notice that taking  $\tau_2^k$  and  $\tau_2^c$  as given is not an innocuous assumption, since that way we avoid confiscatory taxation of the initial wealth. Finally, constraints (15) and (16) guarantee that the policy chosen makes everybody better off than continuing with the status quo policy, guaranteeing a Pareto improving reform. In particular, given that the government objective function does not include the initial generations Equation (15) will always be binding.

Notice that this formulation imposes some restrictions, since the initial generations alive at the beginning of the reform are not part of the objective function, and only appear as a policy constraint. An equivalent formulation would include the initial  $s$  generations in the objective function with a specific weight  $\lambda_s$ , where the weight is chosen to guarantee that the status quo conditions for each generation are satisfied.

The policy maker discounts the future at the exponential rate  $\lambda \in [\underline{\lambda}, 1)$ . The Pareto improving nature of the reform implies that the rate  $\lambda$  has to be big enough to satisfy the participation constraints of all future generations. In particular, if  $\lambda$  were too low then the long run capital stock would be too low and then constraint (16) would be violated in the long run. That restricts the range of admissible values for  $\lambda$  to values where the steady state solution of the government problem for a newborn yields higher utility than in the benchmark economy.

Of course, within a certain range there is some discretionality in the choice of this parameter, implying a different allocation of welfare gains across future generations. We investigate different values for that arbitrary choice.

### Further Constraints on the Ramsey Problem

The main objective of the paper is to identify Pareto improving social security reforms, even though indirectly our exercise also identifies more general Pareto improving tax reforms. When reforming the social security system, relative to our benchmark economy, we want to restrict the government to use the same set of tax instruments. Clearly, moving to a more efficient tax system generates welfare gains, but then it is not clear how much of the welfare gains come from the relaxation of the constraints on the tax instruments.

Notice that the way the Ramsey problem is specified above the fiscal authority has access to age specific taxes.<sup>10</sup> This implies that we provide the fiscal authority with more instruments than in the benchmark economy. In order to prevent that possibility we will introduce additional constraints on the set of instruments. The way to introduce them is to reformulate these constraints in terms of allocations and then impose them as additional constraints on the Ramsey problem.

In particular we will use three types of constraints: age independent labor income taxes, age independent capital income taxes, and finally, a regime where capital income taxes are left unchanged relative to the benchmark.

Constraining taxes to be independent of age implies imposing that the appropriate marginal rate of substitution be the same for all cohorts:

$$\frac{u_{l_{i,t}}}{u_{c_{i,t}} \mathcal{E}_1} = \frac{u_{l_{2,t}}}{u_{c_{2,t}} \mathcal{E}_2} = \dots = \frac{u_{l_{i,t}}}{u_{c_{i,t}} \mathcal{E}_I}, \text{ if } l_{i,t} > 0, t \geq 2 \quad (17)$$

---

<sup>10</sup> Annuities imply that the return on savings must be adjusted for the survival probability. Effectively, then, annuities impose age-specific return on savings. Notice then that annuities are equivalent to the existence of age-specific capital income taxes.

$$\frac{u_{c_{1,t}}}{\varphi_1 u_{c_{2,t+1}}} = \frac{u_{c_{2,t}}}{\varphi_2 u_{c_{3,t+1}}} = \dots = \frac{u_{c_{I-1,t}}}{\varphi_{I-1} u_{c_{I,t+1}}}, \quad t \geq 2 \quad (18)$$

where constraint (17) imposes that labor income taxes are equal for all cohorts working in period  $t$ . Constraint (18) implies that all households pay the same capital income tax and there are no annuity markets. We do so since annuity markets are equivalent to age dependent capital income taxes, and it seems hard to assume that the planner can implement annuities while not being able to discriminate capital income taxes across households.

Finally, if we want capital income taxes to be left unchanged we impose:

$$\frac{u_{c_{1,t}}}{\varphi_1 u_{c_{2,t+1}}} = \frac{u_{c_{2,t}}}{\varphi_2 u_{c_{3,t+1}}} = \dots = \frac{u_{c_{I-1,t}}}{\varphi_{I-1} u_{c_{I,t+1}}} = \frac{\beta}{1+x} \left[ 1 + (1 - \tau^k)(f_{k,t+1} - \delta) \right], \quad t \geq 2 \quad (19)$$

Notice here that whenever we impose equal capital taxes across cohorts we are imposing the first order condition for savings under the assumption of absence of annuities.

Next, Section 6 describes the results for a benchmark case where only labor income taxes equal across cohorts are allowed, i.e. we compute the Ramsey problem including constraints (17) and (19). We take this case as our benchmark since social security financing is associated to labor income taxes. This way we decompose the welfare gains that come directly from the elimination of the distortions most associated to the financing of PAYG systems (i.e. payroll taxes), as compared to the gains coming from the rationalization of other distortions (i.e. age-dependent taxes or capital income taxes). Also, the comparison to the standard literature is more direct, since usually it is only labor income taxes that are changed over the transitions. Later, Section 7 will compare these results with an environment in which the constraints are relaxed.

## 6. Social Security reforms

We focus directly on the design of a Pareto improving transition in an environment where the government is restricted to use a common labor income tax, debt, and one period lump-sum transfers to the initial cohorts as the only fiscal instruments. Hence consumption and capital income taxes are left as in the benchmark economy. Notice that the compensatory transfers could be given to all the initial cohorts and we put no

constraint on their size (except for non-negativity to prevent lump-sum taxation). Since these transfers will have to be paid for with current or future taxes, the fiscal authority will have incentives to use them as little as possible.

Since our main interest is in designing reforms we will focus our attention on the evolution of the policy variables over the transition. In the Appendix we report the evolution of real allocations over time.

We report the evolution of policies over time for three different values of the discount factor in the Ramsey problem: 0.96, 0.97 and 0.98. Values smaller than 0.96 generate less capital accumulation than in the benchmark economy and cannot guarantee that welfare in the final steady state is larger than in the benchmark economy.

In displaying the results we will arbitrarily label the year 2005 to be the Steady State of the benchmark economy and the reform is announced and implemented the following period, i.e. at the beginning of 2010. Remember that a period in the model is 5 years.

The optimally chosen level of transfers to the initial cohorts is reported in Table 2:

<b>Table 2: Transfers to initial generations (% of entitlements)</b>			
Generation	$\lambda = 0.96$	$\lambda = 0.97$	$\lambda = 0.98$
20-24	0.0000	0.0000	0.0000
25-29	0.0000	0.0000	0.0000
30-34	0.0000	0.0000	0.0000
35-39	0.0000	0.0000	0.6005
40-44	0.3365	0.5474	0.8638
45-49	0.6821	0.8030	0.9735
50-54	0.8575	0.9239	1.0132
55-59	0.9413	0.9748	1.0181
60-64	0.9717	0.9860	1.0029
65-69	0.9676	0.9721	0.9769
70-74	0.9986	0.9979	0.9963
75-79	1.0083	1.0048	1.0006
80-84	0.9961	0.9919	0.9877
85-89	0.9745	0.9698	0.9651
TOTAL	0.8926	0.9166	0.9639

Notice that the government chooses to give large compensatory transfers. In total, these transfers amount to a large fraction of total entitlements (89% if  $\lambda = 0.96$ , 92% if  $\lambda = 0.97$  and 96% if  $\lambda = 0.98$ ). The reason is that only changing the future path of labor income taxes it is very difficult to compensate households from the loss of social security pensions, and hence most of the initial generations need to be transferred almost all of their entitlements. In the next section, we will see that this will radically change when the fiscal authority has access to a larger set of policies, in particular age-dependent labor income taxes.

**Figure 3: Evolution of labor income taxes for different  $\lambda$**

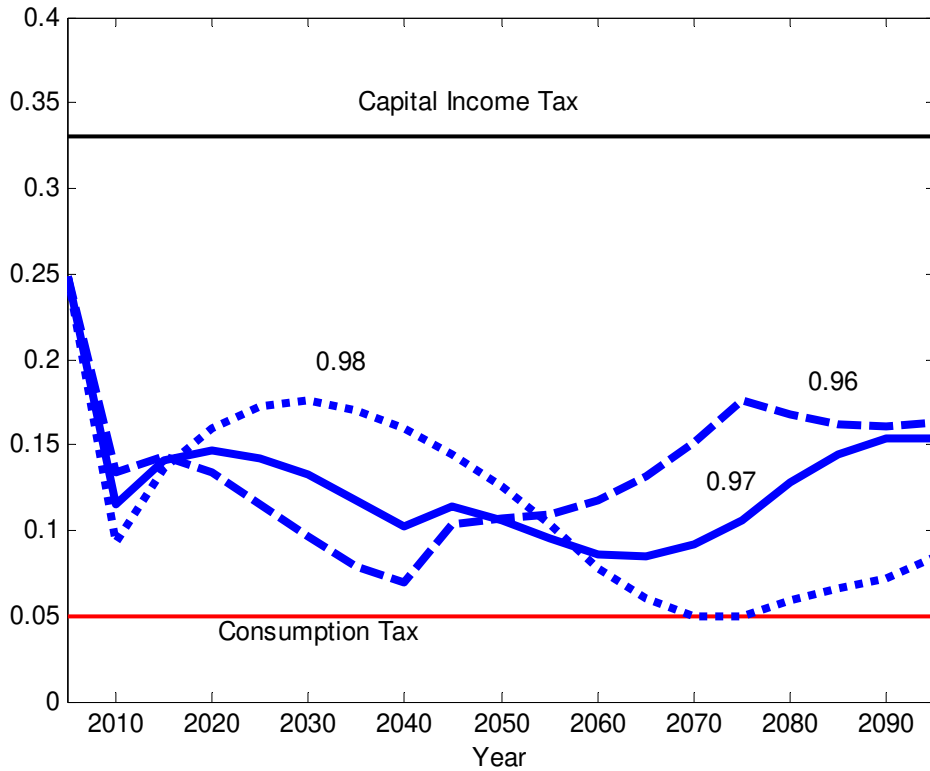
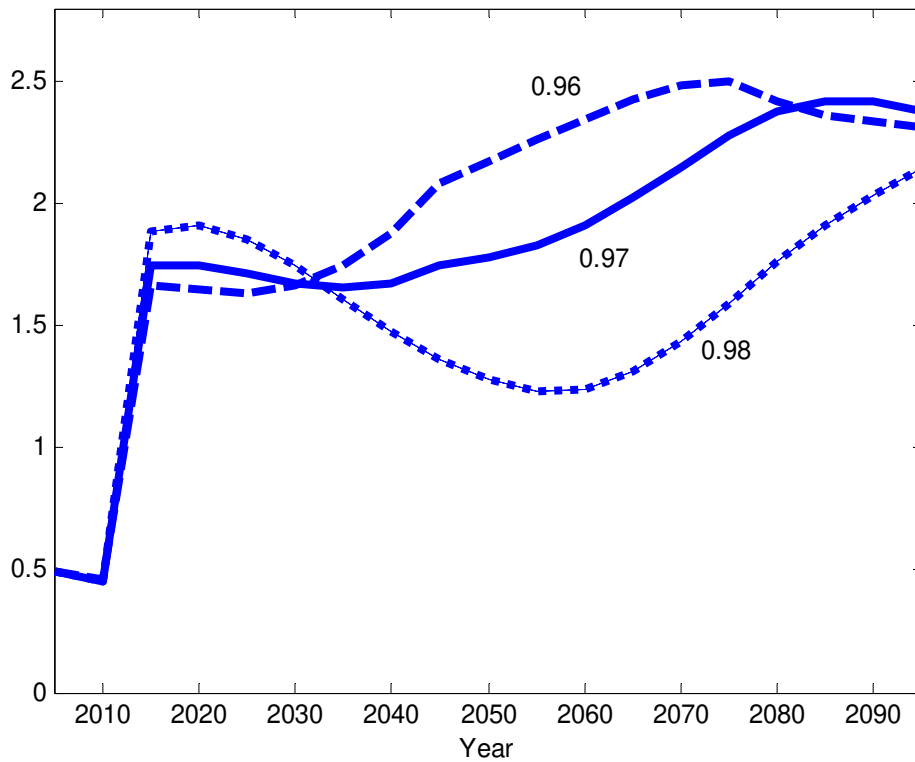


Figure 3 describes the evolution of the optimal labor income tax along the reform. We decentralize the resulting allocation leaving consumption taxes unchanged, even though it is possible to decentralize the same allocation in alternative ways. In particular, we could set consumption taxes to zero and increase labor income taxes so that they are consistent with the optimal wedge chosen by the fiscal authority.

Labor income taxes are substantially lowered the first period following the reform, but then they are increased at different points in time depending on the value of the discount factor. As expected, the more patient the Ramsey planner is, the higher taxes for the transition generations and the lower taxes for the final generations.

The reason why taxes follow such a non-smooth path is that the planner targets different cohorts with only one fiscal instrument. The path of labor income taxes needs to satisfy two objectives: guaranteeing the initial generations the same welfare as in the benchmark economy, and generating welfare gains for subsequent newborns. Since the fiscal authority has to pay a very large amount of compensatory transfers, debt experiences a very large increase on impact. Figure 4 displays the evolution of the debt to GDP ratio for different values of the discount factor.

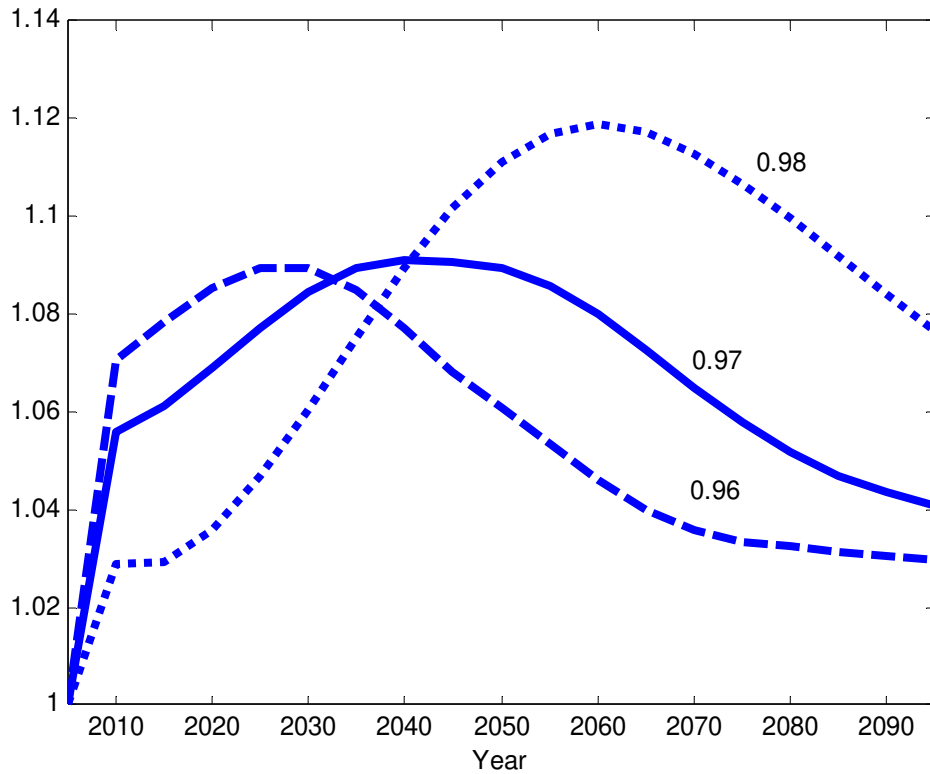
**Figure 4: Evolution of Debt/GDP ratio for different  $\lambda$**



Overall, such a reform generates substantial welfare gains for newborn generations, while leaving cohorts initially alive indifferent by construction. The welfare gains accruing to newborns are plotted in Figure 5.

Notice here very clearly the role of the discount factor in the Ramsey problem. A larger discount factor implies that welfare gains are shifted from early newborns to later newborns. All these welfare numbers are measured in equivalent variation in consumption, so that a value of 1.055 implies that the new generation born in 2010 will experience a welfare increase relative to a newborn in the benchmark economy equivalent to a 5.5% increase in consumption in all periods of their life.

**Figure 5: Evolution of welfare of the newborns for different  $\lambda$**



The welfare gains associated to the reform just discussed are substantial. Measured as equivalent variation in consumption the welfare gains are equivalent to future newborns enjoying between 3 and 8% more consumption than the newborns in the status quo economy with a PAYG social security system.

Such welfare values are lower than most of the welfare gains that the literature has found by comparing steady states with and without a PAYG social security system. The reason is that in our exercise the need to compensate the initial old increases permanently the stock of government debt, so that long run welfare gains are smaller.

## **7. Enlarging the set of fiscal instruments: fiscal reforms**

We just demonstrated how to engineer social security reforms that make everybody better off and lie on the constrained Pareto frontier, keeping the set of fiscal instruments exactly the same as in the benchmark economy and only changing its use over time.



Now we will report the impact of relaxing the constraint on the sets of fiscal instruments.

In what follows we will report all the alternative experiments for the average discount factor we have considered in the previous Section,  $\lambda = 0.97$ . Smaller or larger values would slightly change our conclusions, as we saw in the previous Section.

### **Age-dependent labor income taxes**

Now we turn to the case when the fiscal authority could target labor income taxes directly to different individuals. This is going to be especially important in the initial periods.

In later periods, why would the government choose to tax discriminate? The basic insight is that when individuals exhibit life cycle behavior labor productivity changes with age and the response of consumption, labor and savings decisions to tax incentives varies with age as well, and that might generate incentives to tax discriminate. Erosa and Gervais (2002) or Garriga (1999) explicitly find conditions on preference specifications under which the government would choose to tax discriminate. Garriga (1999) shows that when households' preferences are homothetic in consumption and labor, then the government does not have an incentive to use tax rates that depend upon age, and labor income taxes are uniform for households with an interior solution.

Table 3 reports the optimal amount of compensatory transfers to the initial old. Notice that these are much smaller when the government is allowed to tax discriminate, since the government can compensate through taxes instead of transfers.

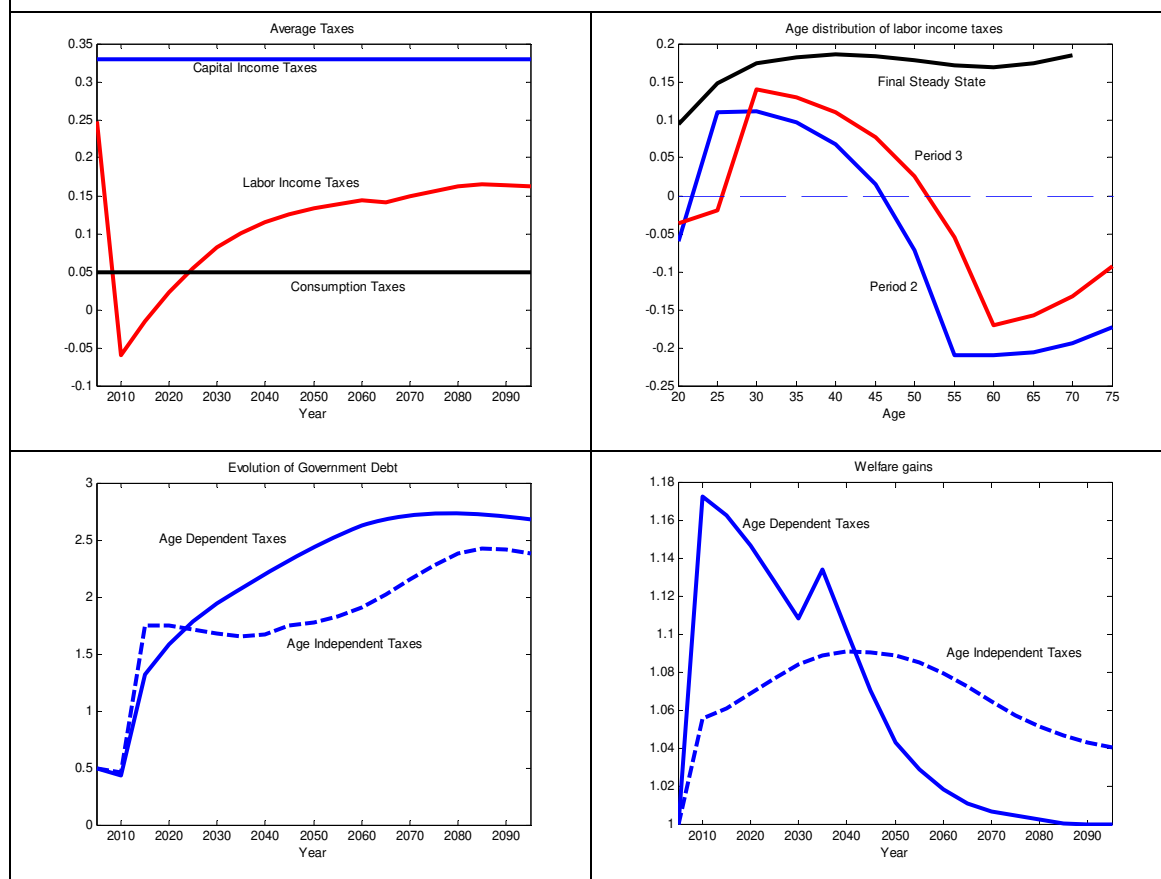
<b>Table 3: Transfers to initial generations (% of entitlements)</b>		
Generation	Age-independent	Age-dependent
20-24	0.0000	0.0000
25-29	0.0000	0.0000
30-34	0.0000	0.0000
35-39	0.0000	0.0000
40-44	0.5474	0.0000
45-49	0.8030	0.0000
50-54	0.9239	0.0000
55-59	0.9748	0.0144

60-64	0.9860	0.4382
65-69	0.9721	0.6828
70-74	0.9979	0.7861
75-79	1.0048	0.8721
80-84	0.9919	0.9499
85-89	0.9698	0.9442
TOTAL	0.9166	0.5187

In particular, with age-dependent labor income taxes only generations after age 55 are compensated through transfers, and in total the government transfers amount only to 52% of the social security entitlements, compared to 92% in the case where taxes have to be the same across cohorts.

On average labor income taxes follow a pattern very similar to the one in the previous Section: a large fall on impact and then taxes are increased progressively. The only difference is that now the path is smoother, since the planner has access to different taxes in order to target different cohorts.

**Figure 6: Optimal policy with age dependent labor income taxes**



How would the government choose to tax discriminate? Figure 6 also reports the shape of the labor income tax schedule for different points in time (upper right panel): the first two periods of the reform (labeled in the figure as Period 2 and Period 3) and the final one (labeled as Final Steady State).

Clearly the extent to which there is tax discrimination is much larger at the beginning of the reform, since the planner wants to use taxes to compensate different cohorts from the loss of social security entitlements. There is still tax discrimination in the final steady state, but the differences in tax rates across cohorts are much smaller than in the initial periods of the reform.

The policy reported generates an increasing path of government debt, slightly similar to the previous one. Also, the value of the social welfare function is higher, since we have just removed one constraint from the maximization problem. However, that does not mean that welfare is uniformly larger for each single newborn at every period. In fact, this alternative policy generates much larger welfare gains for initial newborns, while it does generate very few (if any) welfare gains for future newborns.

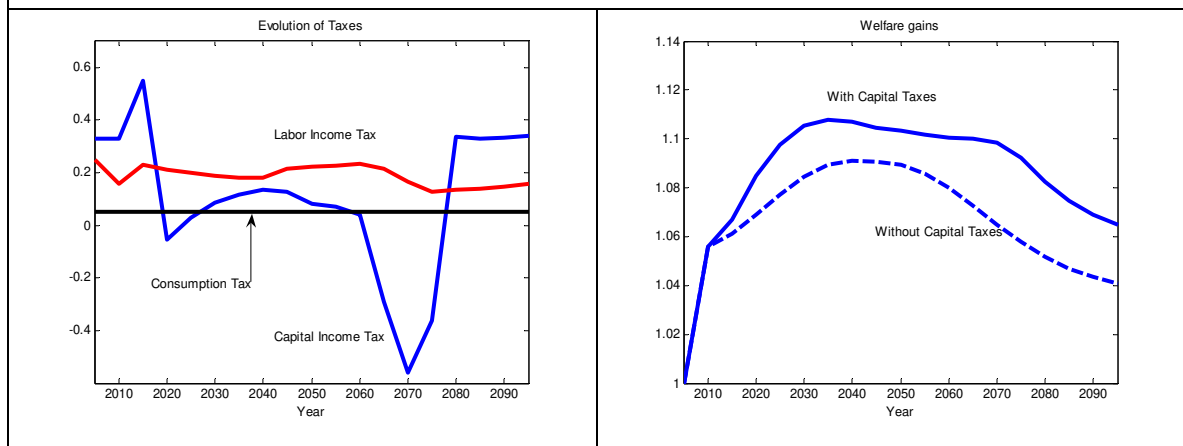
Since the government maximizes a weighted discounted sum of utilities and not a particular cohort, given our choice of the discount factor, it is preferable to front load the welfare gains, provided that the status quo utility constraints are satisfied. In a social security reform where tax rates do not depend on age front loading is very costly. However, with a larger set of instruments this is relatively easier.

### **Capital income taxes**

Now we analyze an environment where the government can also use capital income taxes, but both labor and capital income taxes are constrained to be equal across cohorts. Relative to the benchmark case (only age-independent labor income taxes) the need to resort to initial transfers is almost the same, since the government has to pay 91% of the initial entitlements in transfers (92% in the benchmark economy). The main reason is that the capital stock adjusts slowly over time, so the general equilibrium impact of lower capital income taxes does not substantially benefit initial cohorts.

The resulting policy is a very non-smooth path of both policies, as reported in the first panel of Figure 7. Overall, this policy improves welfare with respect to the case where capital income taxes cannot be used at all (second panel).

**Figure 7: Optimal policy with age independent labor and capital income taxes**



The additional welfare gain is equivalent to a further increase in consumption of 2 percent. This magnitude, and the steady state optimal capital income tax, depends on the choice of  $\lambda$ . For our parameterization, the final steady state capital income taxes are positive and quite large, with a similar magnitude than the benchmark number.

It is important to notice that in the final steady state capital income taxes are positive and quite large. See Conesa, Kitao and Krueger (2007) for a normative analysis of capital income taxation and progressivity in a life-cycle model with different sorts of heterogeneity and risk.

#### **Age-dependent labor and capital income taxes**

Finally, we turn to the case where no constraints are placed on the Ramsey problem, so that the planner can target different cohorts at different points in time with both labor and capital income taxes.

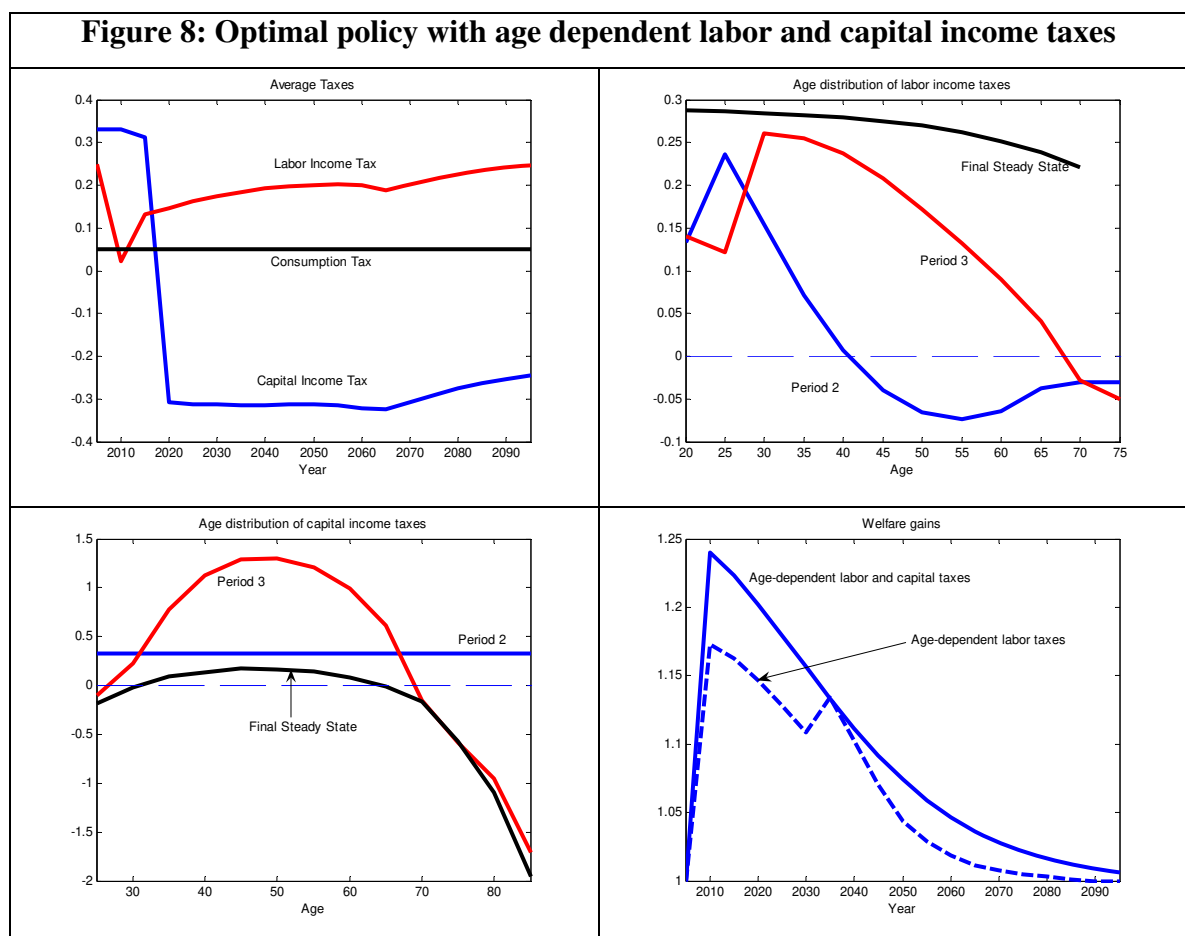
Now the need to resort to compensatory transfers to the initial old is very small, only 13% of the total entitlements. In particular, the government only needs to give transfers to the currently retired, while every other cohort is directly compensated through the appropriate choice of tax rates.

**Table43: Transfers to initial generations (% of entitlements)**

Generation	Benchmark Reform	Age-dependent
20-24	0.0000	0.0000
25-29	0.0000	0.0000

30-34	0.0000	0.0000
35-39	0.0000	0.0000
40-44	0.5474	0.0000
45-49	0.8030	0.0000
50-54	0.9239	0.0000
55-59	0.9748	0.0000
60-64	0.9860	0.0000
65-69	0.9721	0.0000
70-74	0.9979	0.0578
75-79	1.0048	0.2454
80-84	0.9919	0.5312
85-89	0.9698	0.9499
TOTAL	0.9166	0.1309

The evolution of taxes over time for different cohorts and the welfare implications are shown in Figure 8.



Notice that the availability of age-dependent capital income taxes implies the existence of very drastic changes in capital income taxes over time. On average capital income taxes turn into very large subsidies, especially for the oldest cohorts. This way larger welfare gains for the early newborns are possible. It is nevertheless surprising how little difference in welfare gains are attained by introducing such a drastic policy. We interpret this as evidence that the availability of age-dependent labor income taxes does buy a lot in terms of welfare gains.

Effectively, the ability to discriminate capital income taxes across cohorts becomes a very close substitute to compensatory lump-sum transfers to the initial old, and it does not generate substantial additional welfare gains.

## 8. Conclusions

It is a common prediction of standard OG models that changing the PAYG nature of public social security systems towards a FF system might generate substantial efficiency and welfare gains in the long run. However, a common feature in the quantitative evaluation of social security reform proposals is that these long run efficiency and welfare gains come at the cost of substantial welfare losses for transition generations.

We argue that the optimal fiscal policy approach contributes to this literature by maximizing over the entire policy space, rather than comparing exogenously specified policies one to one. Following this approach we identify ways to finance the transition from PAYG to a FF system that lie on the constrained Pareto frontier.

We show that a Pareto neutral reform could be implemented by making explicit the implicit debt of the social security system. However, we quantitatively show that the gains from implementing the optimal fiscal policy, minimizing distortions, are large.

The optimal social security reform consists of providing compensatory transfers to the initial old (almost as large as their social security entitlements) financed with debt and lowering labor income taxes on impact in order to increase them later on.

Investigating the impact of alternative sets of fiscal instruments, we show that the ability to discriminate labor income taxes across cohorts substantially reduces the need to resort to compensatory transfers to the initial old. It also shifts the welfare gains between present and future generations.

Finally, we show that introducing capital income taxes in the analysis allows to generate

some additional welfare gains, but it usually implies very large non-smooth changes in capital income taxes that call into question its relevance as actual policy options.

The same methodology could be adapted to other issues. In a follow-up paper, see Conesa and Garriga (2007), we have used the same methodology to determine the optimal financing of existing pensions in an environment where the economy is subject to a demographic shock.

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## Appendix A: Derivation of the Implementability Constraints

Consider the maximization problem of a household born in period  $t$ :

$$\begin{aligned} \max \quad & \sum_{i=1}^I s_i \beta^{i-1} u(c_{i,t+i-1}, l_{i,t+i-1}) \\ \text{s.t.} \quad & (1 + \tau_{i,t+i-1}^c) c_{i,t+i-1} + (1+x) a_{i+1,t+i} \leq (1 - \tau_{i,t+i-1}^l) w_{t+i-1} \varepsilon_i l_{i,t+i-1} + \left[ 1 + (1 - \tau_{i,t+i-1}^k) r_{t+i-1} \right] a_{i,t+i-1}, \\ & i = 1, 2, \dots, I \end{aligned}$$

Any solution to this maximization problem must satisfy the following conditions:

$$\begin{aligned} [c_{i,t+i-1}] \quad & s_i \beta^{i-1} u_{c_{i,t+i-1}} - \lambda_{i,t+i-1} (1 + \tau_{i,t+i-1}^c) = 0 \\ [l_{i,t+i-1}] \quad & s_i \beta^{i-1} u_{l_{i,t+i-1}} + \lambda_{i,t+i-1} (1 - \tau_{i,t+i-1}^l) w_{t+i-1} \varepsilon_i = 0 \\ [a_{i+1,t+i}] \quad & -(1+x) \lambda_{i,t+i-1} + \lambda_{i+1,t+i} \left[ 1 + (1 - \tau_{i+1,t+i}^k) r_{t+i} \right] = 0 \end{aligned}$$

The standard procedure is to multiply these conditions by the corresponding choice variable, and add up over all periods  $i = 1, 2, \dots, I$ .

So,

$$s_i \beta^{i-1} [c_{i,t+i-1} u_{c_{i,t+i-1}} + l_{i,t+i-1} u_{l_{i,t+i-1}}] = \lambda_{i,t+i-1} [(1 + \tau_{i,t+i-1}^c) c_{i,t+i-1} - (1 - \tau_{i,t+i-1}^l) w_{t+i-1} \varepsilon_i l_{i,t+i-1}]$$

and adding up we obtain the standard Implementability Constraint:

$$\sum_{i=1}^I s_i \beta^{i-1} [c_{i,t+i-1} u_{c_{i,t+i-1}} + l_{i,t+i-1} u_{l_{i,t+i-1}}] = 0$$

Notice that if there are annuities only the budget constraint changes:

$$(1 + \tau_{i,t+i-1}^c) c_{i,t+i-1} + (1+x) \varphi_i a_{i+1,t+i} \leq (1 - \tau_{i,t+i-1}^l) w_{t+i-1} \varepsilon_i l_{i,t+i-1} + \left[ 1 + (1 - \tau_{i,t+i-1}^k) r_{t+i-1} \right] a_{i,t+i-1}$$

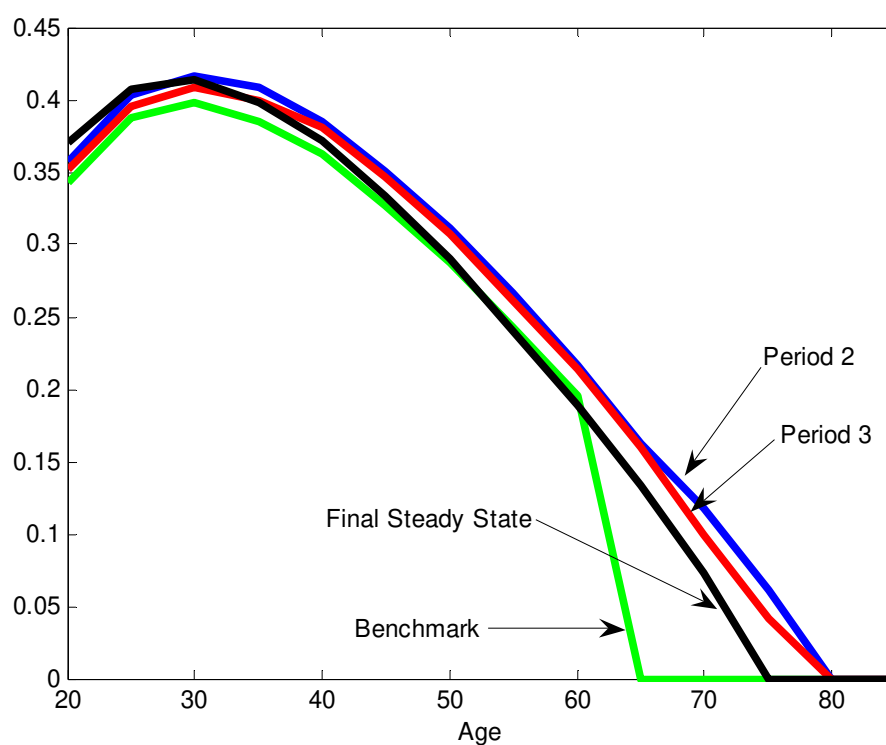
The existence of annuities implies that the need to save for the desired level of future assets is adjusted for survival probabilities. The FOCs would reflect that, but once they are substituted back into the budget constraint they generate exactly the same implementability condition. Also, it is immediate to see that whether taxes are (or not) age dependent does not have any impact on the implementability constraint.

## Appendix B: Evolution of Real Allocations Over the Transition

The allocations in the following figures correspond to the benchmark case with age-independent labor income taxes, leaving capital income taxes unchanged.

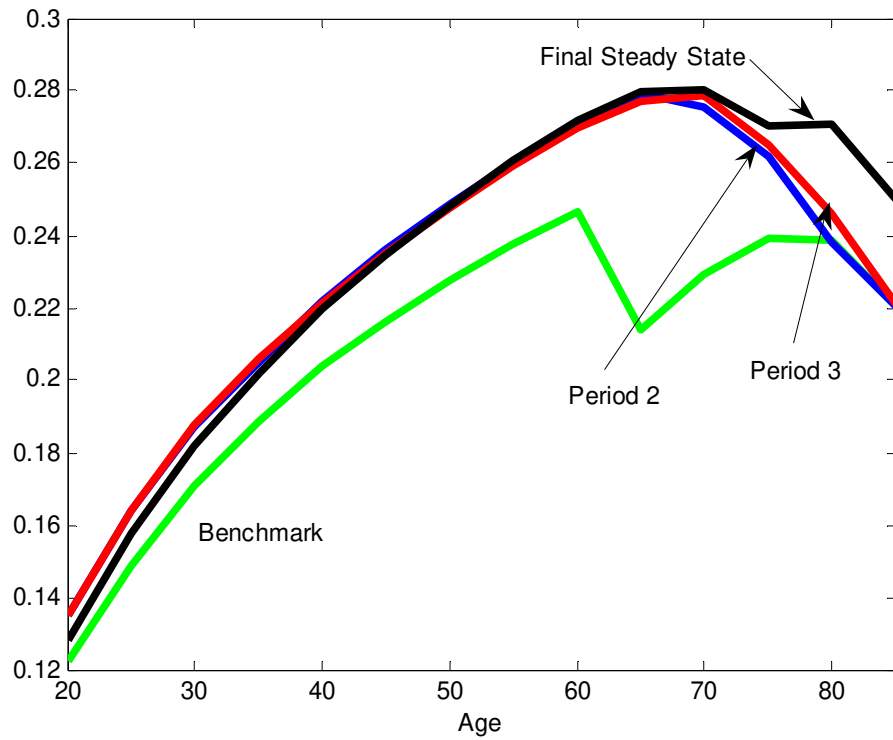
We plot these allocations against those corresponding to the benchmark economy with a PAYG social security system.

**Figure A.1: The Evolution of Hours Worked**



Notice that even though households are allowed to allocate hours to work beyond age 65, the planner sets taxes optimally so that they work very little (or zero).

**Figure A.2: The Evolution of Consumption**



**Figure A.3: The Evolution of the Capital Stock**

